IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Wednesday 8 May 2002 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all five questions from Section A and one question from Section B.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

*Please start each question on a new page. You are advised to show all working, where possible. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Incorrect answers with no working will normally receive **no** marks.*

SECTION A

*Answer all **five** questions from this section.*

1. *[Maximum mark: 13]*

    A construction company wishes to build a new hotel with a maximum of 14 rooms. Market research has shown that the hotel should have at least 3 single rooms and at least 7 double rooms.

    Let x represent the number of single rooms and y represent the number of double rooms.

    (a) Explain why $x + y \leq 14$.   *[1 mark]*

    (b) Write down inequalities in x and y which represent

        (i) the number of single rooms to be built;

        (ii) the number of double rooms to be built.   *[2 marks]*

    (c) (i) On graph paper draw axes, placing the number of single rooms (x) on the horizontal axis and the number of double rooms (y) on the vertical axis. Use the scale 1 cm to represent 1 room on both axes. Clearly label your axes.

        (ii) Represent the inequalities in parts (a) and (b) on your graph.

        (iii) The region R is described by these three inequalities. On your graph shade and label the region R.   *[7 marks]*

    (d) The hotel rooms will cost £80 per single per night and £120 per double per night.

        (i) Write an expression, in terms of x and y, for the nightly revenue n (ie the money taken by the hotel each night).

        (ii) What combination of single and double rooms should be built, so that nightly revenue is maximised?   *[3 marks]*
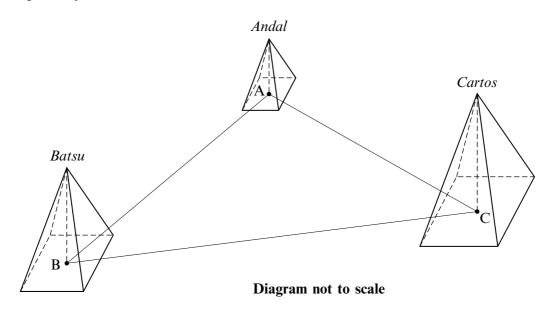
2. *[Maximum mark: 12]*

A National Lottery is offering prizes in a new competition. The winner may choose one of the following.

Option one $1000 each week for 10 weeks.

Option two $250 in the first week, $450 in the second week, $650 in the third week, increasing by $200 each week for a total of 10 weeks.

Option three $10 in the first week, $20 in the second week, $40 in the third week continuing to double for a total of 10 weeks.

(a) Calculate the amount you receive in the tenth week, if you select

  (i) **option two**;

  (ii) **option three**.   *[6 marks]*

(b) What is the total amount you receive if you select **option two**?   *[2 marks]*

(c) Which option has the greatest total value? Justify your answer by showing all appropriate calculations.   *[4 marks]*
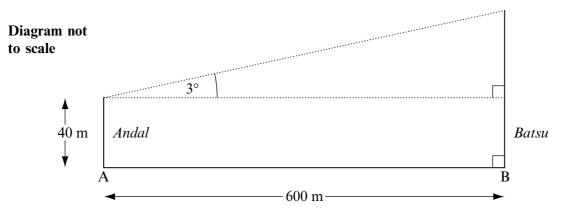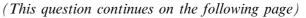
3.   *[Maximum mark: 15]*

Three right pyramids *Andal, Batsu* and *Cartos* were discovered in the dense jungle of *Marhartmasol.* Each pyramid has a square base with centres A, B and C respectively.

Andal

Cartos

Batsu

A

C

B

Diagram not to scale

A surveying team was lowered from a helicopter to the top of *Andal* to take measurements of the area. *Andal* is 40 metres high. The angle of elevation from the top of *Andal* to the top of *Batsu* is 3° . The horizontal distance from A, the centre of the base of *Andal*, to B, the centre of the base of *Batsu* is 600 metres.

(a)   Use the diagram below to find the height of *Batsu*.   *[3 marks]*

**Diagram not
to scale**

3°

40 m   *Andal*   *Batsu*

A   B
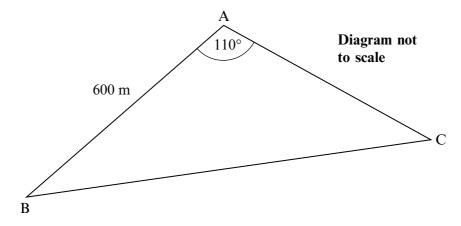
← 600 m →

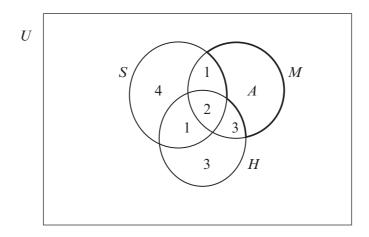*(This question continues on the following page)*

(Question 3 continued)

(b) *Cartos* is found to be 92 metres high and the angle of elevation from the top of *Andal* to the top of *Cartos* is 4° .

(i) Draw a diagram similar to the diagram in part (a) to show the relationship between *Andal* and *Cartos*.

(ii) What is the horizontal distance from A to C?   *[4 marks]*

(c) The diagram below represents measurements relative to the centres of the bases of the pyramids. The surveyors determined the angle at A to be 110° , and the distance AB to be 600 m.

A

110°   **Diagram not
to scale**

600 m

C

B

(i) What is the distance between B and C? Give your answer to the nearest metre.

(ii) What is the size of angle ACB?

(iii) What is the area of the land inside triangle ABC?   *[8 marks]*

4. *[Maximum mark: 15]*

The Venn diagram below shows the number of students studying Science (*S*), Mathematics (*M*) and History (*H*) out of a group of 20 college students. Some of the students do not study any of these subjects, 8 study Science, 10 study Mathematics and 9 study History.

(a)   (i) How many students belong to the region labelled *A*?

     (ii) Describe in words the region labelled *A*.

     (iii) How many students do not study any of the three subjects?   *[5 marks]*

(b) Draw a sketch of the Venn diagram above and shade the region which represents $S' \cap H$.   *[1 mark]*

(c) Calculate $n(S \cup H)$.   *[2 marks]*

(This question continues on the following page)
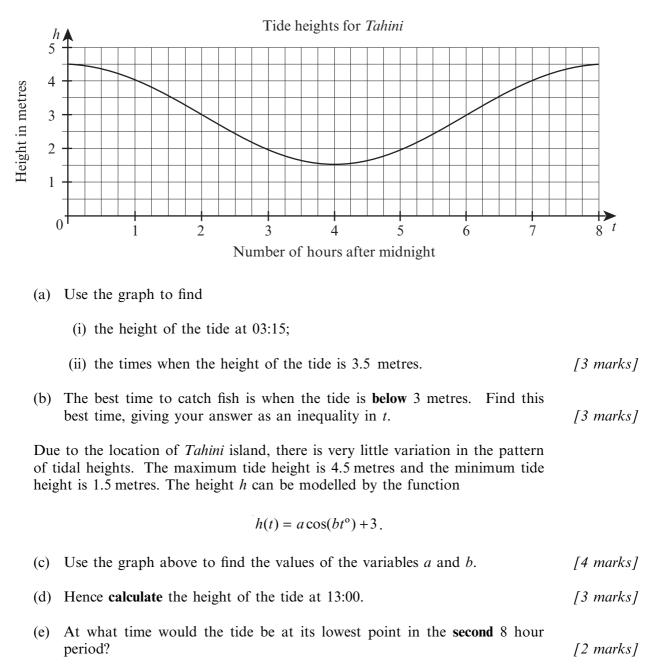
(Question 4 continued)

This group of students is to compete in an annual quiz evening which tests knowledge of Mathematics, Science and History. The names of the twenty students are written on pieces of paper and then put into a bag.

(d) One name is randomly selected from the bag. Calculate the probability that the student selected studies

  (i) all three subjects;

  (ii) History or Science.   *[2 marks]*

(e) A team of two students is to be randomly selected to compete in the quiz evening. The first student selected will be the captain of the team. Calculate the probability that

  (i) the captain studies all three subjects and the other team member does not study any of the three subjects;

  (ii) one student studies Science only and the other student studies History only;

  (iii) the second student selected studies History, given that the captain studies History and Mathematics.   *[5 marks]*

Turn over

5.   *[Maximum mark: 15]*

The graph below shows the tide heights, *h* metres, at time *t* hours after midnight, for *Tahini* island.

Tide heights for *Tahini*

(a)   Use the graph to find

   (i)   the height of the tide at 03:15;

   (ii) the times when the height of the tide is 3.5 metres.   *[3 marks]*

(b)   The best time to catch fish is when the tide is **below** 3 metres.   Find this best time, giving your answer as an inequality in *t*.   *[3 marks]*

Due to the location of *Tahini* island, there is very little variation in the pattern of tidal heights.   The maximum tide height is 4.5 metres and the minimum tide height is 1.5 metres. The height *h* can be modelled by the function

$$h(t) = a\cos(bt°) + 3.$$

(c)   Use the graph above to find the values of the variables *a* and *b*.   *[4 marks]*

(d)   Hence **calculate** the height of the tide at 13:00.   *[3 marks]*

(e)   At what time would the tide be at its lowest point in the **second** 8 hour period?   *[2 marks]*

SECTION B

Answer **one** *question from this section.*

Matrices and Graph Theory

6.   *[Maximum mark: 30]*

   (i) A factory produces two types of washing machines, a front loader (*F*) and
        a top loader (*T*). They supply three shops with washing machines. These
        shops are *Austo* (*A*), *Bolvo* (*B*) and *Catric* (*C*). The number of each type of
        washing machine is shown in the matrix below.

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{pmatrix} F & T \\ 3 & 7 \\ 2 & 5 \\ 8 & 4 \end{pmatrix}$$

   (a) How many front loading washing machines (*F*) are supplied to *Catric*
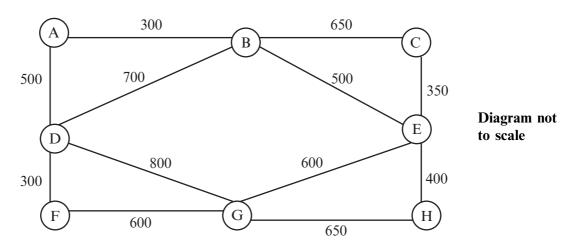   (*C*)?   *[1 mark]*

   (b) The factory sells each front loading washing machine for $*x* and each
        top loading washing machine for $550.

      (i) Construct a 2 by 1 matrix to show the cost of the washing
           machines.

      (ii) Write down a matrix expression to represent the cost of the
           washing machines for the three shops.

      (iii) If *Bolvo* spent a total of $3440 on washing machines, what is the
            cost of one front loading washing machine?   *[4 marks]*

(This question continues on the following page)

(Question 6 continued)

(ii) The graph below shows the network of streets and intersections in the main shopping area of a small town. The vertices, labelled with capital letters, represent the intersections, and the edges represent the streets. The distances shown on the graph are in metres.

Diagram not to scale

(a) (i) Give a path that goes from the intersection F to the intersection E.

   (ii) What is the degree of vertex D?

   (iii) Construct a simple, connected subgraph containing the vertices A, B, C, D and E. *[3 marks]*

Traffic lights are to be installed at each intersection. All the lights are to be linked together by a system of cables. The cables are to be laid under the existing streets.

(b) Draw a diagram to show how the traffic lights can be connected using the minimum amount of cable. *[2 marks]*

(c) A special machine is used to clean the streets. It can clean both sides of the street at the same time.

   (i) What is the minimum distance the machine has to travel in order to clean all the streets and return to the point where it started?

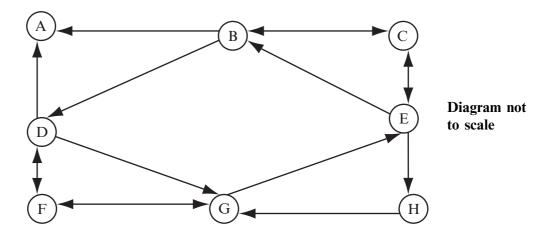   (ii) Give **one** route the machine could take, starting and finishing at H. *[3 marks]*

(This question continues on the following page)

(Question 6 (ii) continued)

(d) The council has redesigned the street system in a series of one-way and two-way streets, as shown below.

Diagram not to scale

The information represented in the graph is also represented in the adjacency matrix (R) below.

$$
\begin{array}{c c}
 & \text{TO} \\
 & \begin{array}{cccccccc}
\text{A} & \text{B} & \text{C} & \text{D} & \text{E} & \text{F} & \text{G} & \text{H}
\end{array}
\end{array}
$$

		A	B	C	D	E	F	G	H
	A	0	0	0	0	0	0	0	0
	B	1	0	1	x	0	0	0	0
F	C	0	1	0	0	1	0	0	0
R	D	1	0	0	0	y	1	1	0
O	E	0	1	1	0	0	0	0	1
M	F	0	0	0	1	0	0	1	0
	G	0	0	0	0	1	1	0	0
	H	0	0	0	0	0	0	1	0

(i) Find the values of x and y.

(ii) List all the paths which go from intersection E to intersection D, without passing through any vertex more than once.

(iii) What is the significance of the first row of zeros?

(iv) Suggest **one** change you would make to the graph above to improve the traffic flow.

[6 marks]

(This question continues on the following page)

(Question 6 continued)

(iii) An army reserve unit has organised a mock battle between two teams on a deserted island. The mission of team *Alpha* is to attack the island and the mission of team *Omega* is to defend the island. The island is only accessible from the north or the south.

The team with the most points will win the battle. The table below shows the possible points for team *Alpha*. For example, if team *Alpha* attacks from the north and team *Omega* defends the north, team *Alpha* will lose 200 points.

| | | Team *Omega* | |
		North	South
	North	–200	300
Team *Alpha*			
	South	800	–300

Both teams want to use a strategy that will minimise their losses (*play safe strategy*).

(a) (i) What is the play safe strategy for team *Omega*?

   (ii) What is the play safe strategy for team *Alpha*?

   (iii) What is the result if both teams adopt their play safe strategy?

   (iv) If team *Omega* assumes that team *Alpha* will adopt their play safe strategy on the first attack, which position should they defend? Explain your choice.   *[5 marks]*

Several attacks have to be made before either team wins. Team *Omega* uses a mixed strategy. They defend the north against a proportion p of the attacks. The matrix product below gives the expected points for team *Alpha* in attacking either from the north or the south, when team *Omega* uses the mixed strategy.

$$\begin{pmatrix} -200 & 300 \\ 800 & -300 \end{pmatrix} \begin{pmatrix} p \\ 1-p \end{pmatrix}$$

(b) What does $1-p$ represent?   *[1 mark]*

(c) Write an expression in terms of p that represents the expected points, if team *Alpha* attacks from

   (i) the north;

   (ii) the south.   *[2 marks]*

(This question continues on the following page)

(Question 6 (iii) continued)

(d) (i) Show that the value of p which gives the most effective play safe mixed strategy for team *Omega* is $p = \dfrac{3}{8}$.

(ii) For this value of p, how many points can team *Alpha* expect to win at each attack? *[3 marks]*
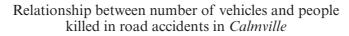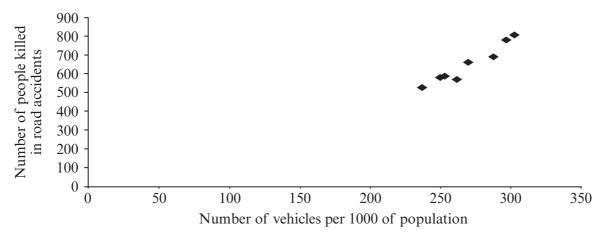
Turn over

Further Statistics and Probability

7.   *[Maximum mark: 30]*

(i) The scatter diagram below shows the relationship between the number of vehicles per thousand of population and the number of people killed in road accidents over an eight year period in *Calmville*.

Relationship between number of vehicles and people
killed in road accidents in *Calmville*

Let *x* be the number of vehicles per thousand and *y* be the number of people killed. The following information is known.

$$\bar{x} = 270, \quad \bar{y} = 650, \quad s_x = 22.3, \quad s_y = 96.2, \quad s_{xy} = 2077.75$$

(a)   (i) Calculate the product–moment correlation coefficient (*r*).

(ii) Explain clearly the statistical relationship between the variables *x* and *y*.   *[4 marks]*

(b) Write the equation of the regression line of *y* on *x*, expressing it in the form $y = mx + c$ (where *m* and *c* are given correct to 3 significant figures).   *[4 marks]*

(c) Use your equation in part (b) to answer the following questions.

(i) There were 250 vehicles per 1000 of population. Find the number of people killed.

(ii) Explain why it is not a good idea to use the regression line to estimate the number of people killed when the number of vehicles is 150 per thousand.   *[3 marks]*

(ii) A survey was conducted in a company to determine whether position in upper management was independent of gender. The results of this survey are tabulated below.

	Managers	Junior executives	Senior executives	Totals
Male	95	130	75	300
Female	65	110	25	200
Totals	160	240	100	500

The table below shows the expected number of males and females at each level, if they were represented proportionally to the total numbers of males and females employed.

	Managers	Junior executives	Senior executives	Totals
Male	a	c	60	300
Female	b	d	40	200
Totals	160	240	100	500

(a) (i) Show that the expected number of **Male Managers** (a) is 96.

(ii) Hence find the values of b, c and d. *[5 marks]*

(b) (i) Write a suitable null hypothesis for this data.

(ii) Write a suitable alternate hypothesis for this data. *[2 marks]*

(c) (i) Perform a chi-squared test of independence for this data to show the value of χ^2 is 12.8 to 3 significant figures.

(ii) Calculate the number of degrees of freedom, and write down the critical value of χ^2 at the 5% significance level.

(iii) What conclusion can be drawn regarding gender and position in upper management? *[6 marks]*

(iii) The working life of a television set is normally distributed with mean 6000 hours and standard deviation of 1500 hours.

(a) The average television operates for approximately 1825 hours per year. What percentage of televisions is expected to break down in the first 12 months? *[2 marks]*

(b) It is known that 10% of the televisions have a lifetime of n hours (ie break down after n hours). Find the value of n. *[4 marks]*
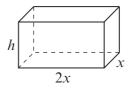
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Introductory Differential Calculus

8.　*[Maximum mark: 30]*

(i)　(a)　Let $y = x^3 + 3x^2 + 3x + 4$.

　　　　(i)　Find $\dfrac{dy}{dx}$.

　　　　(ii)　Find the gradient of the curve when $x = -2$.

　　　　(iii)　Is the function y increasing or decreasing when $x = -2$?

　　　　(iv)　Find the value(s) of x when the gradient of the curve is 3.

　　　　(v)　Find the x-coordinate of the point of inflexion.　　　*[11 marks]*

　　(b)　Given that $f(x) = 2x + 3$, find $f'(x)$ using

$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right).$$ 　　*[3 marks]*

　　(c)　The function $g(x)$ has gradient function $g'(x) = 3x^2 - x + 1$.　Find
$g(x)$, given that $g(2) = 3$.　　　　　　　　　　　　*[3 marks]*

(ii)　The length of the base of a cuboid is twice the width x , and its height is
h centimetres, as shown in the diagram below.　Its total surface area is
A cm^2 and its volume is V cm^3 .

　　(a)　Show that $A = 4x^2 + 6xh$.　　　　　　　　　　　　*[2 marks]*

　　(b)　If $A = 300$, find an expression for h , in terms of x .　　*[2 marks]*

　　(c)　If $A = 300$, show that the volume $V(x) = 100\, x - \dfrac{4}{3} x^3$.　　*[2 marks]*

　　(d)　(i)　Find $V'(x)$.

　　　　(ii)　Solve $V'(x) = 0$.

　　　　(iii)　Find the maximum volume.

　　　　(iv)　What is the height of the cuboid when the volume is maximised?　*[7 marks]*